

Axion string cosmology and its controversies

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Abstract.

Understanding axion cosmology has important experimental consequences since it constrains the range of allowed values for the axion mass. In the standard thermal scenario, which assumes Peccei-Quinn symmetry restoration after inflation, an axion string network forms at the phase transition $T \sim f_a$ and then radiatively decays into a cosmological background of axions. Under standard assumptions for the evolution of this string network and the radiation from it, axions must have a mass $m_a \sim 100 \mu\text{eV}$ with specified large uncertainties. We discuss critically the various suggestions in the literature that the axion mass might be lighter.

1. Introduction

The axion has remained a popular dark matter candidate because of its enduring motivation as an elegant solution to the strong CP-problem [1]. Despite early hopes of discovery, it turned out that in order to be consistent with accelerator searches and astrophysics, the axion must be nearly ‘invisible’ and extremely light. Its couplings and mass are inversely proportional to the (large) Peccei-Quinn (PQ) scale f_a as in

$$m_a = 6.2 \times 10^{-6} \text{eV} \left(\frac{10^{12} \text{GeV}}{f_a} \right). \quad (1)$$

Accelerator constraints have been largely superseded by those from astrophysics; because the axion is so weakly coupled, volume effects can compete with other surface and convective stellar energy loss mechanisms. The

strongest astrophysical constraints on the axion mass derive from studies of supernova 1987a and conservative estimates yield $m_a \lesssim 10 \text{ meV}$ [2]. The present programme of large-scale axion search experiments [3] are sensitive to a mass range $m_a \sim 1\text{--}10 \mu\text{eV}$, which has been chosen for a variety of historical and technological reasons. Here, we discuss the motivation from cosmology for a larger mass axion ($m_a \sim 100 \mu\text{eV}$) if it forms a substantial part of the dark matter of the universe. We will also comment on suggestions in the literature that question this point of view.

2. Standard axion cosmology

The cosmology of the axion is determined by the two energy scales f_a and Λ_{QCD} . The first important event is the PQ phase transition which is broken at a high temperature $T \sim f_a \gtrsim 10^9 \text{ GeV}$. This creates the axion, at this stage an effectively massless pseudo-Goldstone boson, as well as a network of axion strings [4] which decays gradually into a background cosmic axions [5]. (Note that one can engineer models in which an inflationary epoch interferes with the effects of the PQ phase transition, this possibility is discussed in ref. [6]). At a much lower temperature $T \sim \Lambda_{\text{QCD}}$ after axion and string formation, instanton effects ‘switch on’, the axions acquire a small mass, domain walls form [7] between the strings [4] and the complex hybrid network annihilates into axions in about one Hubble time [8].

There are three possible mechanisms by which axions are produced in the ‘standard thermal scenario’, which assumes PQ symmetry restoration after any inflationary epoch: (i) thermal production, (ii) axion string radiation and (iii) hybrid defect annihilation when $T = \Lambda_{\text{QCD}}$. Axions consistent with the astrophysical bounds must decouple from thermal equilibrium very early; their subsequent history and number density is analogous to the decoupled neutrino, except that unlike a 100eV massive neutrino, thermal axions cannot hope to dominate the universe with $m_a \lesssim 10 \text{ meV}$. We now turn to the two dominant axion production mechanisms, but first we address an important historical digression.

2.1. Misalignment misconceptions

The original papers on axions suggested that axion production primarily occurred, not through the above mechanisms, but instead by ‘misalignment’ effects at the QCD phase transition [9]. Before the axion mass ‘switches on’, the axion field θ takes random values throughout space in the range 0 to 2π ; it is the phase of the PQ-field lying at the bottom of a $U(1)$ ‘Mexican hat’ potential. However, afterwards the potential becomes tilted and the true minimum becomes $\theta = 0$, so the field in the ‘misalignment’ picture begins to coherently oscillate about this minimum; this homogeneous mode

corresponds to the ‘creation’ of zero momentum axions. Given an initial rms value θ_i for these oscillations, it is relatively straightforward to estimate the total energy density in zero momentum axions and compare these to the present mass density of the universe (assuming a flat $\Omega = 1$ FRW cosmology) [9, 10]:

$$\Omega_{\text{a,hom}} \approx 2 \Delta h^{-2} \theta_i^2 f(\theta_i) \left(\frac{10^{-6} \text{eV}}{m_{\text{a}}} \right)^{1.18} \quad (2)$$

where $\Delta \approx 3^{\pm 1}$ accounts for both model-dependent axion uncertainties and those due to the nature of the QCD phase transition, and h is the rescaled Hubble parameter. The function $f(\theta)$ is an anharmonic correction for fields near the top of the potential close to unstable equilibrium $\theta \approx \pi$, that is, with $f(0) = 1$ at the base $\theta \approx 0$ and diverging logarithmically for $\theta \rightarrow \pi$ [6]. If valid, the estimate (2) would imply a constraint $m_{\text{a}} \gtrsim 5 \mu\text{eV}$ for the anticipated thermal initial conditions with $\theta_i = \mathcal{O}(1)$ [9, 10].

As applied to the thermal scenario, the expression (2) is actually a very considerable underestimate for at least two reasons: First, the axions are not ‘created’ by the mass ‘switch on’ at $t = t_{\text{QCD}}$, they are already there with a specific momentum spectrum $g(k)$ determined by dynamical mechanisms prior to this time. The actual axion number obtained from $g(k)$ is much larger than the rms average assumed in (2) which ignores the true particle content. Secondly, this estimate was derived before much stronger topological effects were realized, notably the presence of axion strings and domain walls. In any case, these nonlinear effects complicate the oscillatory behaviour considerably, implying that the homogeneous estimate (2) is poorly motivated.

2.2. Axion string network decay

Axions and axion strings are inextricably intertwined. Like ordinary superconductors or superfluid ^4He , axion models contain a broken $U(1)$ -symmetry and so there exist vortex-line solutions. Combine this fact with the PQ phase transition, which means the field is uncorrelated beyond the horizon, and a random network of axion strings must inevitably form. An axion string corresponds to a non-trivial winding from 0 to 2π of the axion field θ around the bottom of its ‘Mexican hat’ potential. It is a global string with long-range fields, so its energy per unit length μ has a logarithmic divergence which is cut-off by the string curvature radius $R \lesssim t$, that is, $\mu \approx 2\pi f_{\text{a}}^2 \ln(t/\delta)$, where the string core width is $\delta \approx f_{\text{a}}^{-1}$. The axion string, despite this logarithmic divergence, is a strongly localized object and is likely to behave as such; if we have a string stretching across the horizon at the QCD temperature, then $\ln(t/\delta) \sim 65$ and over 95% of its energy lies within a tight cylinder enclosing only 0.1% of the horizon volume. To first order, then, the string behaves like a local cosmic string, a

fact that can be established by a precise analytic derivation and careful comparison with numerical simulations [11].

After formation and a short period of damped evolution, the axion string network will evolve towards a scale-invariant regime with a fixed number of strings crossing each horizon volume (for a cosmic string review see ref. [12]). This gradual demise of the network is achieved by the production of small loops which oscillate relativistically and radiate primarily into axions. The overall density of strings splits neatly into two distinct parts, long strings with length $\ell > t$ and a population of small loops $\ell < t$, that is, $\rho = \rho_\infty + \rho_L$. High resolution numerical simulations confirm this picture of string evolution and suggest that the long string density during the radiation era is $\rho_\infty \approx 13\mu/t^2$ [13]. To date, analytic descriptions of the loop distribution have used the well-known string ‘one scale’ model, which predicts a number density of loops defined as $\mu \ell n(\ell, t) d\ell = \rho_L(\ell, t) d\ell$ in the interval ℓ to $\ell + d\ell$ to be given by

$$n(\ell, t) = \frac{4\alpha^{1/2}(1 + \kappa/\alpha)^{3/2}}{(\ell + \kappa t)^{5/2} t^{3/2}}, \quad (3)$$

where α is the typical loop creation size relative to the horizon and $\kappa \approx 65/[2\pi \ln(t/\delta)]$ is the loop radiation rate [14]. Once formed at $t = t_0$ with length ℓ_0 , a typical loop shrinks linearly as it decays into axions $\ell = \ell_0 - \kappa(t - t_0)$. The key uncertainty in this treatment is the loop creation size α , but compelling heuristic arguments place it near the radiative backreaction scale, $\alpha \sim \kappa$. (If this is the case, we note that the loop contribution is over an order of magnitude larger than direct axion radiation from long strings.)

String loops oscillate with a period $T = \ell/2$ and radiate into harmonics of this frequency (labelled by n), just like other classical sources. Unless a loop has a particularly degenerate trajectory, it will have a radiation spectrum $P_n \propto n^{-q}$ with a spectral index $q > 4/3$, that is, the spectrum is dominated by the lowest available modes. Given the loop density (3), we can then calculate the spectral number density of axions $dn_a/d\omega$, which turns out to be essentially independent of the exact loop radiation spectrum for $q > 4/3$. From this expression we can integrate over ω to find the total axion number at the time t_{QCD} , that is, when the axion mass ‘switches on’ and the string network annihilates. Subsequently, the axion number will be conserved, so we can find the number-to-entropy ratio and project forward to the present day. Multiplying the present number density by the axion mass m_a yields the overall axion string contribution to the density of the universe [14]:

$$\Omega_{\text{a,string}} \approx 110 \Delta h^{-2} \left(\frac{10^{-6} \text{eV}}{m_a} \right)^{1.18} f(\alpha/\kappa), \quad (4)$$

where

$$f(\alpha/\kappa) = \left[\left(1 + \frac{\alpha}{\kappa} \right)^{3/2} - 1 \right]. \quad (5)$$

The key additional uncertainty from the string model is the ratio $\alpha/\kappa \sim \mathcal{O}(1)$, which should be clearly distinguished from particle physics and cosmological uncertainties inherent in Δ and h (which appear in all estimates of Ω_a). With a Hubble parameter near $h = 0.5$, the string estimate (4) tends to favour a dark matter axion with a mass $m_a \sim 100\mu\text{eV}$, moreover a comparison with (2) confirms that $\Omega_{a,\text{string}}$ is well over an order of magnitude larger than the ‘misalignment’ contribution.

2.3. Hybrid defect annihilation

Near the QCD phase transition the axion acquires a mass and network evolution alters dramatically because domain walls form. Large field variations around the strings collapse into these domain walls, which subsequently begin to dominate over the string dynamics. This occurs when the wall surface tension σ becomes comparable to the string tension due to the typical curvature $\sigma \sim \mu/t$. The demise of the hybrid string–wall network proceeds rapidly, as demonstrated numerically [8]. The strings frequently intersect and intercommute with the walls, effectively ‘slicing up’ the network into small oscillating walls bounded by string loops. Multiple self-intersections will reduce these pieces in size until the strings dominate the dynamics again and decay continues through axion emission.

An order-of-magnitude estimate of the demise of the string–domain wall network indicates that there is an additional contribution [15]

$$\Omega_{a,\text{dw}} \sim \mathcal{O}(10)\Delta h^{-2} \left(\frac{10^{-6}\text{eV}}{m_a} \right)^{1.18}. \quad (6)$$

This ‘domain wall’ contribution is ultimately due to loops which are created at the time $\sim t_{\text{QCD}}$. Although the resulting loop density will be similar to (3), there is not the same accumulation from early times, so it is likely to be subdominant [14] relative to (4). More recent work, [16] questions this picture by suggesting that the walls stretching between long strings dominate and will produce a contribution anywhere in the wide range $\Omega_{a,\text{dw}} \sim (1\text{--}44)\Omega_{a,\text{string}}$; however, this assertion requires stronger quantitative support. Overall, like most effects,¹ the domain wall contribution will serve to further strengthen the string bound (4) on the axion.

¹ We note briefly that it is also possible to weaken any axion mass bound through catastrophic entropy production between the QCD-scale and nucleosynthesis, that is, in the timescale range $10^{-4}\text{s} \lesssim t_{\text{ent}} \lesssim 10^{-2}\text{s}$. Usually this involves the energy density of the universe becoming temporarily dominated by an exotic massive particle with a tuned decay timescale.

Up to this point we have only considered the simplest axion models with a unique vacuum $N = 1$, so what happens when $N > 1$? In this case, any strings present become attached to N domain walls at the QCD-scale. Such a network ‘scales’ rather than annihilates, and so it is cosmologically disastrous being incompatible (at the very least) with CMB isotropy.

3. Theoretical challenges to $m_a \sim 100\mu\text{eV}$

The conclusion that that m_a is most likely around $100\mu\text{eV}$ is an important one since the only experimental search for axions with a realistic level of sensitivity can only detect axions if $m_a \sim 1 - 10\mu\text{eV}$. The possibility of detection by the current generation of experiments might still be possible, either by pushing the theoretical and cosmological uncertainties of the calculation presented in the previous section to the limit, allowing for entropy production by a late decaying massive particle, or by allowing an inflationary epoch which prevents PQ symmetry restoration². However, the theoretical framework which we have presented in the previous section has been also challenged conceptually and quantitatively in the literature and here we shall comment critically on this work.

The basis of our understanding global string evolution and radiation presented in the previous section is based on the assumption that global strings behave in an essentially similar way to local strings described by the an extension of the Nambu action, known as the Kalb-Ramond action. This kind of action is used frequently in superstring theory to describe the dynamics of strings coupled to a massless scalar field, and its efficacy for describing global string dynamics was established quantitatively in ref. [11]. Since the global strings have long range forces and a logarithmically divergent energy density, one might wonder how this is possible. But as we have already pointed out over 95% of the energy lies within 0.1% of the volume for cosmological scales. For the point of view of this discussion we will separate the challenges in the literature into the conceptual [17] and the quantitative [18]. Both are just supported by numerical simulations, rather than the analytic arguments and numerical calculations used in our description. The main point being that use of numerical simulations of limited physical size and dynamic range can be hazardous when making extrapolations to cosmological scales where the strength of the relevant effects is very much diminished.

² We should note that as discussed in ref. [6], even if the inflationary reheat temperature is below f_a , it is still possible for the strings to be formed and for them to be the dominant source of axions.

3.1. Conceptual issues : the spectrum of radiation from strings

While the issue of the spectrum of radiation from global strings is a rich and interesting theoretical subject, its effects on the cosmological axion density are much simpler than often realised. The main issue, as elucidated in ref. [19], relates to the power in high frequency modes. The frequency of the n th radiation mode for a loop is $\omega_n = 4\pi n/l$, where l is its length. If the fall-off of the power spectrum of radiation is a power law ($P_n \propto n^{-q}$ for large n , as suggested by analytic calculations using the Kalb-Ramond action [20], then so long as $q > 1$ the calculation we presented in the previous section is still valid. This is because only in the divergent case $q = 1$ does the calculation of the axion density depend sensitively on the high frequency cut-off [17]. A wide range of analytic and numerical calculations support this issue, but since this issue has been discussed extensively in a large number of publications, we see no reason to discuss it further here.

3.2. Quantitative issues : the scaling density of strings

Recent simulations of a network of global strings on a ‘large’ (256^3) grid has suggested that the scaling density of strings $\xi = \rho_\infty t^2 \mu \approx 1$, rather than $\xi \approx 13$ as used in our calculation. This clearly has a significant effect on the axion mass prediction, reducing it to around $10\mu eV$, since the axion density depends linearly on ξ . The argument that $\xi \approx 13$ is based on the premise that the cosmological scale global strings behave in an essentially similar way to the Nambu strings for the reasons already discussed, and this may have to be modified to account for any slight deviations from this. However, in the simulations presented in ref. [18] there is no reason to believe that the strings will behave anything like Nambu strings because $\log(t/\delta)$ is at most 4, that is, in the same 0.1% of the volume of space, there is less than 50% of the energy. The long range forces between strings will be more than 10 times stronger than in the cosmological context and will substantially effect the dynamics and hence the scaling density.

Furthermore, the dynamic range of the simulations is much smaller than it appears. The time of formation of the string network is $t_f = 20t_i$, where t_i is the start of the simulation, and the time of the end of the simulation is $t_e = 80t_i$. Hence, in real time t the dynamic range is $t_e/t_f = 4$, not 80 as one might think, and the true dynamic range in conformal time is only $\tau_e/\tau_f = (t_e/t_f)^{1/2} = 2$. Therefore, the simulations are too small to simulate realistic cosmological behaviour and have very limited dynamic range.

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